MODEL-BASED CONTROL OF SHAKE TABLES WITH MULTI-METRIC FEEDBACK

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ABSTRACT

Shake tables provide a direct means by which to evaluate structural performance under earthquake excitation. Because the entire structure is mounted on the table and subjected to the ground motion in real-time, dynamic effects and rate-dependent behavior can be represented. Shake table control is not straightforward as the desired signal is an acceleration record while actuators operate in displacement feedback for stability. At the same time, the payload is typically massive relative to the capacity of the actuator, leading to pronounced control-structure interaction. Through this interaction, the dynamics of the specimen influence the dynamics of the shake table, which can be additionally problematic when specimens change behavior due to damage or other nonlinearities. Moreover, shake tables are inherently nonlinear, making it difficult to accurately recreate a desired acceleration record over a broad frequency range. A model-based multi-metric feedback control strategy is proposed to improve tracking of the desired acceleration, remaining robust to nonlinearities including changes in specimen condition. The proposed strategy is verified for the shake table testing of both linear and nonlinear specimens.

Keywords: Shake table testing, actuator control, multi-metric feedback

1. INTRODUCTION

Shake table testing provides a direct means by which to excite a physical specimen with a desired ground motion. Because the entire structure is mounted on the table, dynamic effects can be directly captured. The payload (including table mass) is typically large relative to the capacity of the actuator, leading to pronounced control-structure interaction (CSI). Through this interaction, the dynamics of the specimen influence the dynamics of the shake table (Dyke *et al.*, 1995), which can be additionally problematic when specimens change behavior due to damage or other nonlinearities. Moreover, shake tables are inherently nonlinear, making it difficult to accurately recreate a desired acceleration record over a broad frequency range. Shake table control is not straightforward as the desired signal is an acceleration record. Since acceleration measurements (i.e. from accelerometers) cannot capture constant velocities or constant displacements, acceleration feedback alone cannot provide stable control. For stability, even in shake table applications, actuators typically operate in displacement feedback through an inner-loop PID controller.

Spencer and Yang (1998) present the transfer function iteration method used for many commercial shake tables. This approach is based on a linearized model of the shake table dynamics from command signals to measured accelerations. An inverse of this model is used to generate a command signal history from the acceleration record, taking into account the modeled table behavior. However, nonlinearities lead to error between desired and measured accelerations. These errors are used offline to iteratively modify the command signal. If the specimen undergoes damage or the shake table operates outside of the linearized range, this approach may lead to acceleration tracking errors.

The proposed model-based multi-metric feedback control strategy is designed to improve tracking of the desired acceleration, while remaining robust to nonlinearities including changes in specimen conditions. The proposed strategy is experimentally verified with the shake table testing of both linear and nonlinear specimens. To ensure stability of the shake table, the inner-loop displacement feedback control will not be modified. Rather, model-based feedforward and feedback links will be added as an outer-loop controller to improve acceleration tracking.

2. MODEL-BASED CONTROL

The model-based control strategy proposed herein is based on a linearized model of the shake table dynamics. The control strategy is an application of model-based control developed for real-time hybrid simulation in Phillips and Spencer (2011). The input-output relationship between command to the shake table u and measured displacements x_m and measured accelerations a_m can be represented by the following transfer function:

$$\mathbf{G}_{yu}(s) = U(s)^{-1}\begin{bmatrix} X(s) \\ A(s) \end{bmatrix} = \frac{\mathbf{Y}(s)}{U(s)} \tag{1}$$

A state-space representation of Eqn. (1) is given by:

$$\dot{\mathbf{z}} = \mathbf{A}\mathbf{z} + \mathbf{B}u \tag{2}$$

$$\mathbf{y} = \mathbf{C}\mathbf{z} \tag{3}$$

where $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ are the state, input, and output matrices, respectively, $\mathbf{z}$ is the state vector, and $\mathbf{y} = \{x_\mathrm{m} \quad a_\mathrm{m}\}^\mathrm{T}$. Regulator redesign can be used to create a model-based feedforward-feedback controller to minimize the tracking error defined as:

$$\mathbf{e} = \mathbf{r} - \mathbf{y} \tag{4}$$

where $\mathbf{r} = \{x_\mathrm{d} \quad a_\mathrm{d}\}^\mathrm{T}$ and x_d and a_d are the desired displacement and acceleration, respectively. The command u should be chosen such that the tracking error is minimized. If perfect tracking is achieved, an ideal state $\bar{\mathbf{z}}$ and an ideal input $\bar{u}$ leading to an output $\bar{\mathbf{y}}$ must exist such that $\bar{\mathbf{y}} = \mathbf{r}$. Deviations of the state, control, and output from this ideal system with respect to the original system are defined as:

$$\tilde{\mathbf{z}} = \mathbf{z} - \bar{\mathbf{z}} \tag{5}$$

$$\tilde{u} = u - \bar{u} \tag{6}$$

$$\tilde{\mathbf{y}} = \mathbf{y} - \bar{\mathbf{y}} \tag{7}$$

The dynamics of the deviation system are then:

$$\dot{\tilde{\mathbf{z}}} = \mathbf{A}\tilde{\mathbf{z}} + \mathbf{B}\tilde{\mathbf{u}} \tag{8}$$

$$\tilde{\mathbf{y}} = \mathbf{C}\tilde{\mathbf{z}} = -\mathbf{e} \tag{9}$$

The tracking problem has now been redefined as a regulator problem about a set-point (Lewis and Syrmos, 1995). The control law in Eqn. (6) can be rewritten in terms of the original system, which consists of a feedforward component $\bar{u} = u_{\mathrm{FF}}$ determined from the ideal system and a feedback component $\tilde{u} = u_{\mathrm{FB}}$ determined from the deviation system, i.e.,

$$u = \bar{u} + \tilde{u} = u_{\mathrm{FF}} + u_{\mathrm{FB}} \tag{10}$$

The multi-metric model-based controller is presented in Fig. 1.

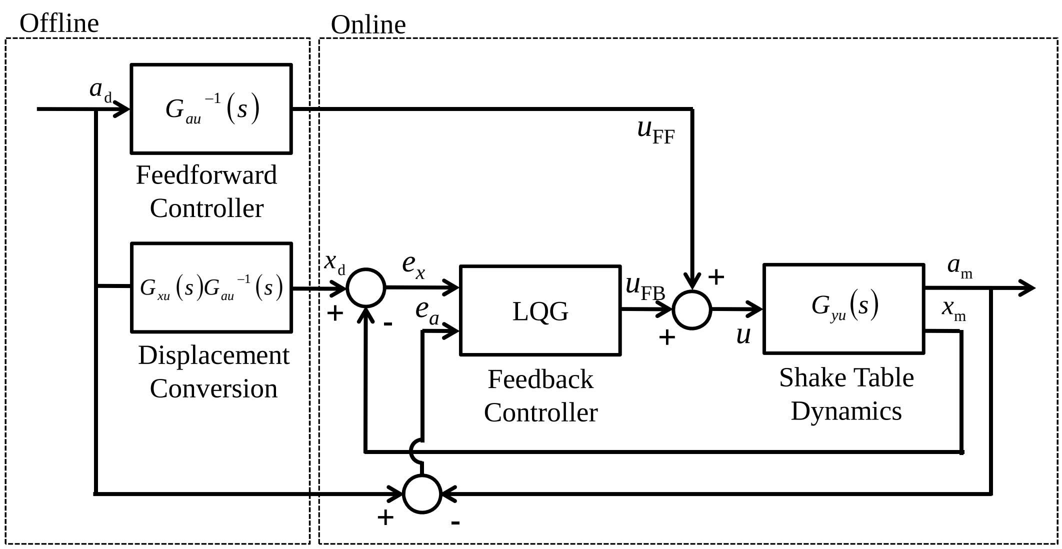

Figure 1. Model-based multi-metric feedback control

The feedforward controller is designed to cancel out the dynamics of the shake table. The feedforward controller is based on the model considering only acceleration as in:

$$G_{\mathrm{FF}}(s) = G_{au}(s)^{-1} = \frac{U(s)}{A(s)} \tag{11}$$

The feedback controller is added to complement the feedforward controller, adding robustness in the presence of changing specimen conditions, modeling errors, and disturbances. For the proposed model-based feedback controller, linear-quadratic-Gaussian (LQG) control is applied to bring the deviation states to zero and thus reduce the tracking error. Both displacement and acceleration measurements are used for better estimates of the states of the shake table system model through a Kalman Filter (Kalman, 1960), improving the LQG control. Displacement measurements are more sensitive in the lower frequency range while acceleration measurements are more sensitive in the higher frequency range; thus, by combining the two measurements, accurate feedback control can be achieved over a broad frequency range. To attenuate the control effort at frequencies beyond the region of desired tracking performance, the LQG controller process noise is shaped by a second-order filter.

To implement multi-metric feedback control, the desired displacement is required. The displacement can be calculated using the feedforward controller (acceleration to command) and the shake table model (command to displacement) as in Eqn. (12) (Nakata, 2010).

$$G_{xu}(s)G_{au}^{-1}(s)=\frac{X(s)}{U(s)}\frac{U(s)}{A(s)}=\frac{X(s)}{A(s)} \tag{12}$$

3. EXPERIMENTAL STUDY

The model-based control strategy for shake table testing is verified using a small-scale single axis shake table as shown in Fig. 2. The shake table uses a custom built servo-motor manufactured by SMI Technology to move a 46 cm × 46 cm top plate with a stroke of ±5 cm. A Quanser Consulting MultiQ-3 Board and host PC are used to control the shake table in displacement control with a PD controller. Accelerations are measured using model 3701G3FA3G capacitive accelerometers manufactured by PCB Piezotronics. The accelerometers have a measurement range of ±3 g and a frequency range of 0-100 Hz. Model-based outer-loop controllers are implemented using a dSPACE model 1103 DSP board.

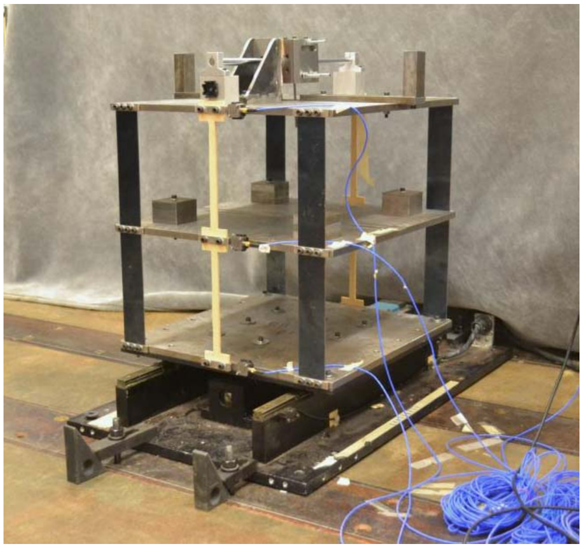

Figure 2. Shake table with two-story structure

A two-story steel frame structure is added to the shake table to study tracking performance in the presence of CSI. The building consists of 1.27 cm thick steel plates connected by spring-steel columns that constrain the motion to a single axis, minimizing torsion. To investigate controller performance in the presence of specimen damage, both linear (steel columns only) and nonlinear (steel columns plus wooden columns, see in Fig. 2) are considered. The natural frequencies of the structure are 1.67 Hz and 4.63 Hz without wooden columns and 1.97 Hz and 5.49 Hz with wooden columns.

3.1. Linear Specimen

The input-output relationship of the shake table is determined experimentally using a 0 to 20 Hz BLWN command to the shake table. Four levels of excitation are considered, 0.1, 0.2, 0.4, and 0.6 V RMS, to investigate the influence of excitation amplitude on the shake table dynamics. Transfer functions are shown in Fig. 3, which display the input command to displacement and acceleration transfer functions. The units for command, measured displacement, and measured accelerations in the transfer function are

Volts, cm, and m/s^2. It is clear that the shake table exhibits highly nonlinear behavior due to the amplitude dependency of the transfer functions. At low amplitudes, there is significant friction limiting the accurate tracking of the command signal. Also, the two natural frequencies of the structure are apparent in the shake table transfer function. Due to the pronounced CSI, two transfer function modeling approaches are considered. The first modeling approach is a low-order model, where the peaks and valleys due to CSI are ignored. The second modeling approach is a high-order model, where a greater number of poles and zeros are added to model CSI accurately. Both low and high-order model fits are illustrated in Fig. 3. Since the transfer function at 0.4 V RMS represents amplitude similar to that of the reference ground motions, the model-based controller will be based on this level of excitation. By design, the feedback controller will make the system robust to nonlinearities, modeling inaccuracies, and changes in the system.

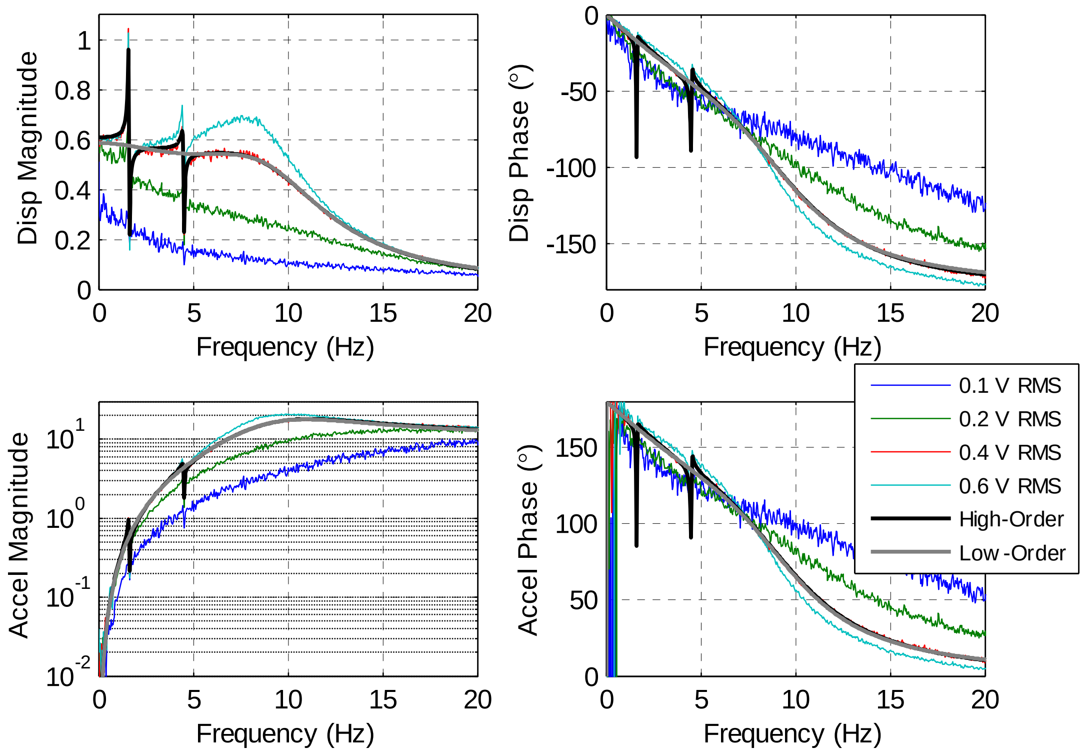

Figure 3. Shake table transfer function with linear structure

Reference earthquake ground motions are selected from a benchmark study on structural control (Ohtori *et al.*, 1994). These include: the NS component of the El Centro Earthquake of 1940, the NS component of the Sylmar County Hospital parking lot during the Northridge Earthquake of 1994, and the NS component of the Japanese Meteorological Agency station during the Kobe Earthquake of 1995.

The performance of the shake table with the linear building is investigated for model-based controllers based on both low and high-order models. Controllers include feedforward control (FF) and feedforward control with multi-metric feedback (FF + FB). A 15 second window of the results for Kobe earthquake record is shown in Fig. 4. The accelerations are filtered using a low-pass filter with a cutoff at 15 Hz.

Multi-metric feedback improves the performance of acceleration tracking compared to feedforward control alone, especially in reducing higher frequency oscillations and matching the peak accelerations. The best feedback controller designs were achieved by placing more importance on the acceleration measurement relative to the displacement measurement (through LQG design), illustrating the value of acceleration feedback. Furthermore, the controller developed using the high-order model performs better than the lower-order model. If the frequency content of the input ground motion overlaps with the pronounced CSI effects observed in the transfer functions, the higher-order model will provide better control over this region. The results for all earthquake records investigated, summarized by the RMS error, are presented in Table 1. Results from the other earthquake records confirm the excellent tracking performance of the model-based multi-metric feedback control. Again, the controllers based on higher-order models provide better control in most cases.

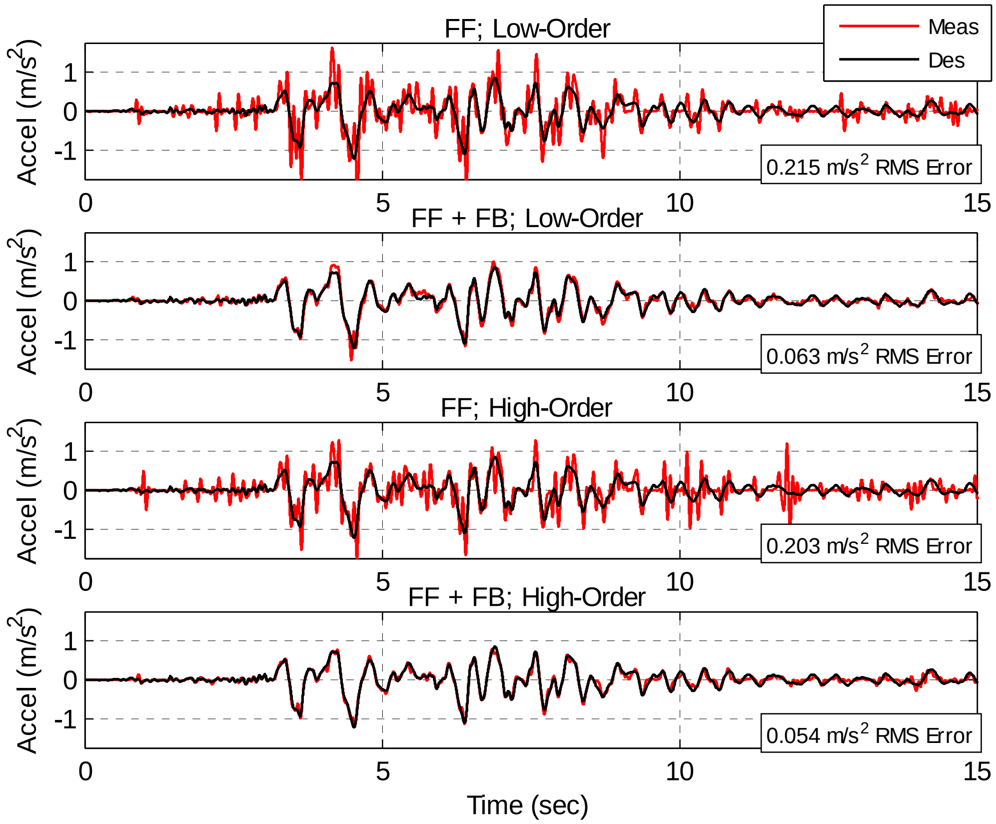

Figure 4. Tracking of 0.15x Kobe with linear structure

TABLE 1. TRACKING PERFORMANCE FOR CONTROLLERS

Earthquake Record	RMS Error (m/s^2)			
	Low-Order Controllers		High-Order Controllers	
	FF	FF + FB	FF	FF + FB
0.2 x El Centro	0.177	0.049	0.157	0.049
0.4 x El Centro	0.259	0.065	0.220	0.058
0.15 x Kobe	0.215	0.063	0.203	0.054
0.1 x Northridge	0.153	0.057	0.126	0.031

3.2. Nonlinear Specimen

System identification is repeated with wooden columns installed using the same testing protocol as in the linear case for 0.4 V RMS excitation. The columns were not damaged during system identification. Fig. 5 shows the command to displacement and acceleration transfer functions. Also, the linear specimen transfer functions from Fig. 3 are shown to illustrate the change that the additional columns introduce to the dynamics of the shake table. Both low-order and high-order model fits are shown.

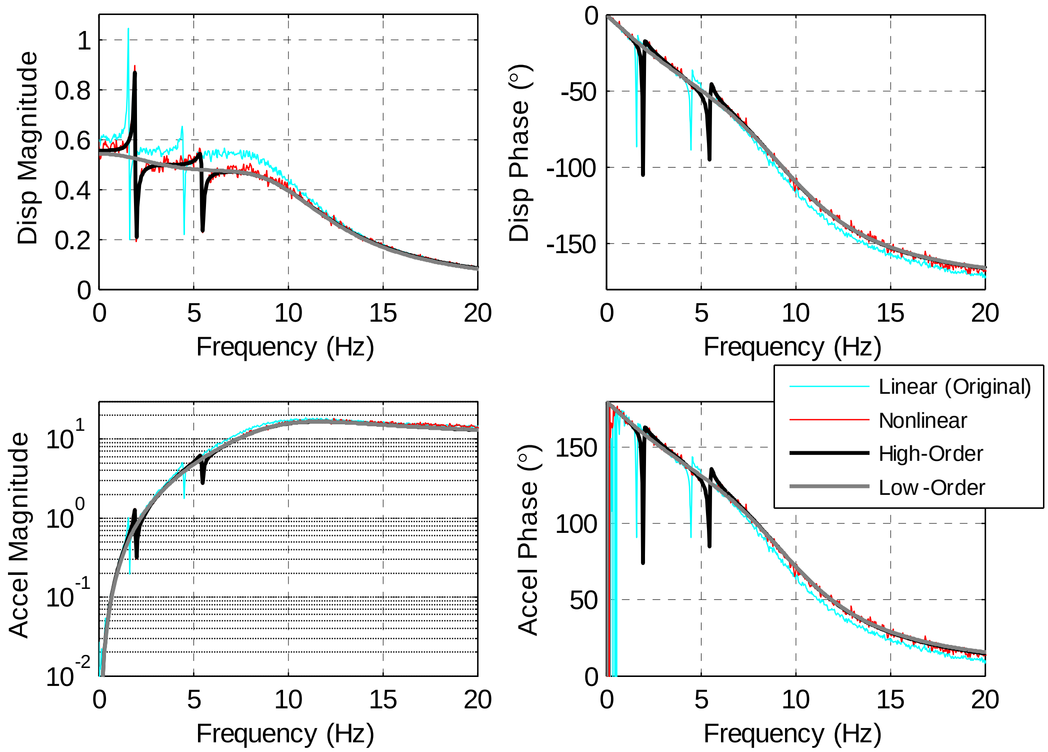

Figure 5. Shake table transfer function with nonlinear structure

The El Centro earthquake record (0.4 x) is selected as the reference acceleration. As the specimen will undergo damage during the earthquake, experiments are not easily repeated; therefore, only the model-based multi-metric feedback controller will be explored for both low-order and high-order models. Results from the experiments, each starting with undamaged wooden columns which become damaged during testing, are shown in Fig. 6. As the conditions of the specimen change due to damage, the high-order controller is no longer tuned, and performance degrades slightly compared to tests with the linear structure. The low-order model controller is less sensitive to changes in the natural frequency, thus as the condition of the specimen does not affect the controller performance as significantly.

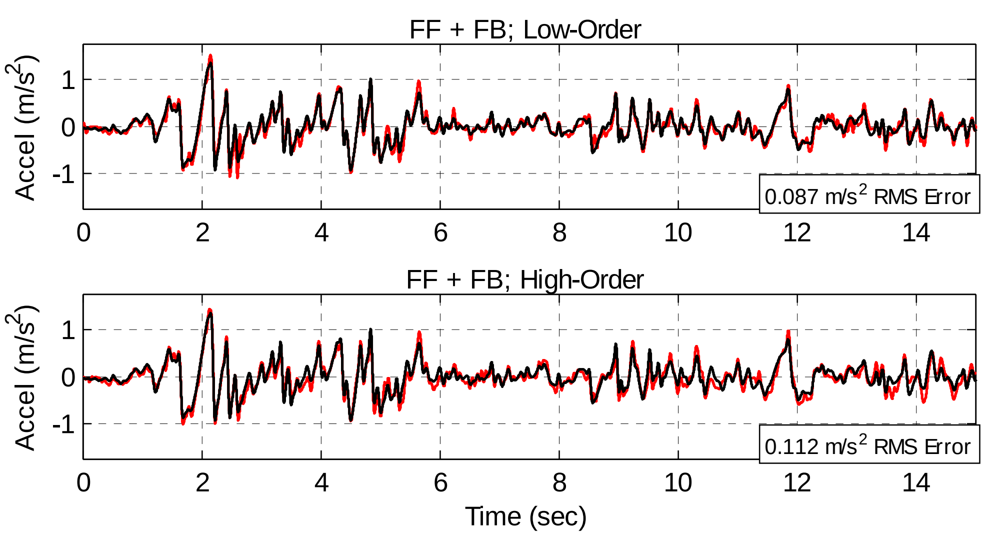

Figure 6. Tracking of 0.15x Kobe with nonlinear structure

4. CONCLUSIONS

In this research, the proposed model-based controller was applied to shake table testing. High-fidelity tracking of the desired accelerations was achieved through a multi-metric feedback approach. Success was achieved by ensuring shake table stability through inner-loop displacement feedback control while designing the model-based controller as an outer-loop controller. Also, the flexibility of LQG control for model-based feedback allows for shaping of the process and measurement noises. Through shaping filters, the feedback controller can be designed to be less sensitive to higher frequencies which otherwise may lead to high frequency oscillations and instabilities.

The model-based controller was demonstrated to be robust to changes in specimen conditions due to damage. In cases when damage is expected, the low-order controller less precisely tuned to the peaks and valleys due to CSI was shown to provide better control over all specimen conditions. For linear specimens, the high-order controller exactly tuned to the effects of CSI was shown to provide better control. In either case, model-based control provides an excellent alternative to existing control techniques for shake table testing. Further improvement is expected when working with higher-quality hydraulic shake tables where the behavior will be less amplitude dependent.

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